



Implementation of McEliece Algorithm in Code-Based Cryptography

Erikson Putra Perdana Lumbantobing¹, Githa Sabrina Pasaribu², Nitamasi Finowa³, Oktoma Jaya Halawa⁴, Lucanhi Situmeang⁵

Department of Informatics Engineering, Faculty of Computer Science Universitas Katolik Santo Thomas Medan Jl. Setia Budi No.479-F, Tanjung Sari, Medan Selayang District, Medan City, North Sumatra 20132, Indonesia

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ABSTRACT

The McEliece algorithm is an asymmetric cryptosystem based on error-correcting codes, relying on the complexity of the syndrome decoding problem for its security. This study discusses the implementation of the McEliece algorithm using the Hamming(7,4) code in the encryption and decryption process of binary messages. Encryption is done by generating a public key consisting of a disguised generator matrix G' , a permutation matrix P , and a non-singular matrix SSS . The binary message is encrypted by adding controlled noise to increase security. In the decryption phase, the received message is processed using reverse permutation and error detection with a parity check matrix to recover the original message. Experiments are carried out by implementing the algorithm in Python, with results showing successful encryption and decryption of messages according to the McEliece theoretical framework. This study confirms that the Hamming code can be used as a simplified approach to the implementation of McEliece, although with security limitations compared to Goppa codes.

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Corresponding Author:

Githa Sabrina Pasaribu

E-mail:

kuliahgithasabrina@gmail.com

1. INTRODUCTION

Various cryptographic techniques have been developed to ensure secure communication, one of which is code-based cryptography. The McEliece algorithm, introduced by Robert McEliece in 1978, is an asymmetric cryptosystem that utilizes error-correcting codes to encrypt and decrypt messages. Unlike conventional cryptographic algorithms such as RSA and ECC, which rely on number theory problems, McEliece is based on the complexity of syndrome decoding, making it a strong candidate for quantum computing attacks.(Parpunguan & Panjaitan, 2024.).

One of the main features of the McEliece algorithm is the use of linear error correcting codes to secure the messages sent.(Widyawati & Utomo, 2024).By adding controlled noise during encryption, the algorithm increases security while ensuring that the message can still be recovered by an authorized recipient. In this study, the Hamming(7,4) code is used as the basis for implementing the McEliece algorithm because of its structural simplicity and error-correcting capabilities.(Anggraini, 2024).

This study aims to explore the implementation of McEliece cryptosystem using Hamming(7,4) code, analyze its encryption and decryption processes, and evaluate its effectiveness in maintaining data security. This study further involves the development of a Python-based implementation to demonstrate the practical application of this cryptographic method.

2. METHOD

This study uses an experimental approach to implement and evaluate the McEliece cryptographic algorithm using the Hamming(7,4) code. The methodology is structured as follows:

Key Making

Key generation is the first and most important step in McEliece's algorithm, which involves the creation of private and public keys. The private key consists of three components: the generating matrix

Implementation of McEliece Algorithm in Code-Based Cryptography. Erikson Putra Perdana Lumbantobing, et.al

G, the non-singular matrix S, and the permutation matrix P. The public key is derived from the private key using the following formula:

$$G' = S \cdot G \cdot P$$

- G : The Hamming(7,4) code generator matrix, which is used to map a 4-bit message into a 7-bit code word.
- S : A randomly generated non-singular matrix (4x4) is used to randomize the password.
- Q : A permutation matrix (7x7) that permutes the bits of the encoded message.
- Public Key Generation:** The public key G' is calculated by multiplying the matrices S, G, and P. The public key is used for encryption, while the private key is kept secret and used for decryption.

Encryption Process

The encryption process involves transforming a plain text message into cipher text that is secure for transmission.

- Message Conversion**
The message to be encrypted is first converted into binary form using ASCII encoding. For example, the character "Z" (ASCII 90) is represented in binary as 01011010. This binary message is then divided into 4-bit blocks because the Hamming(7,4) code operates on 4-bit messages.
- Public Key Encoding** For each 4-bit message block, the corresponding 7-bit codeword is obtained by multiplying the message block by the public key matrix G' : The formula for encoding each block is:

$$c. \quad I \cdot G' = ci$$

Where:

- I is the i -th 4-bit message block.
- ci is the corresponding 7-bit code word (block of cipher text).
- Error Recognition** To improve security, noise vectors e are introduced to simulate errors during transmission. The noise vectors are randomly generated and added to the ciphertext. For example, an error vector of size 7 (equal to the length of the password) could be [0, 0, 0, 0, 1, 0, 0], which modifies the ciphertext. The final ciphertext is:

$$b) \quad y = c + e$$

Where y is the final ciphertext with additional errors.

Decryption Process

The decryption process is the reverse of encryption and involves recovering the original message from the cipher text.

- Receiving Ciphertext**
The recipient receives the ciphertext y , which consists of a 7-bit code word with potential errors.
- Inverse Permutation** The first step in decryption is to invert the permutation applied during encryption. This is done by multiplying the ciphertext by the inverse of the permutation matrix P^{-1} . The formula is:
$$y' = y \cdot P^{-1}$$
where y' is the transformed ciphertext after the inverse permutation is applied.
- Error Detection and Correction**
After inverting the permutation, the parity check matrix H is used to detect and correct any errors. The syndrome is calculated as:
$$s = y' \cdot HT$$
If the syndrome is not zero, an error is present, and the error bit position can be identified and corrected using the syndrome information.
- Decoding**
Once the errors are corrected, the final step is to decode the corrected message. The corrected codeword is multiplied by the inverse of the S^{-1} matrix to obtain the original 4-bit message block. The formula for decoding is:

$$m = c' \cdot S^{-1}$$

Where m is the recovered binary message.

- Message Conversion Back**

The last 4-bit block is then converted back to ASCII representation and recombined to form the original message.

3. RESULTS AND DISCUSSION

This section presents the results obtained from the implementation of the McEliece cryptosystem using the Hamming(7,4) code. The discussion includes key generation, encryption, decryption, error correction performance, and an analysis of the system's effectiveness in handling errors. The results are validated by comparing the decrypted messages with the original plaintext messages and evaluating the impact of the introduced errors.

McEliece's algorithm is a code-based cryptography method that uses linear code for encryption and decryption. Here are the basic formulas and steps in this algorithm:

Key Making

$$G' = S \cdot G \cdot P$$

G : The generating matrix of the Goppa code (dimensions $k \times n$).

S : A random non-singular matrix of size $k \times k$.

P : A permutation matrix of size $n \times n$. The public key is calculated as:

Encryption Process

A binary message m (a vector of length k) is encrypted using the public key as follows:

Public Generator Matrix

- Use the public generator matrix G' to convert the message into a password before adding noise.
- Initial encryption formula

$$G' = S \cdot G \cdot P \quad c' = m \cdot G'$$

Where:

c' is the ciphertext before noise is added.

G' is a disguised public generator matrix, obtained from:

G is the original generator matrix of the Hamming(7,4) code.

S is a non-singular matrix of size $k \times k$.

P is a permutation matrix of size $n \times n$.

Add Noise e to Ciphertext

- To increase security, an error vector e is added to the ciphertext.
- The error vector e is chosen so that it has exactly t bits set to 1, for a total length of n .
- Final encryption formula:

$$y = c' + e$$

Where:

- y is the final ciphertext sent.
- e is an error vector that causes disturbances in the ciphertext.

Manual Calculation

The process of decrypting encrypted messages using the McEliece cryptography system with the Hamming code.

McEliece Decryption Steps with Hamming Code (7,4)

- Converting the Character "Z" to Binary. The character "Z" has an ASCII value of 90. Converting 90 to binary gives: $90_{10} = 01011010_2$. The binary vector representation: $m = [0 \ 1 \ 0 \ 1 \ 1 \ 0 \ 1 \ 0]$ is
- Key Preparation. Before encryption, we need to create public and private keys. Choose Hamming Code (7,4). Code word length: $n = 7$ bits. Message length: $k = 4$ bits. Error correction capability: 1 bit. Generator Matrix (G). Hamming code (7,4) has the following generator matrix G :

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

3. Permutation Matrix (P). A permutation matrix is used to shuffle the bits in a code word.
Example:

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

4. Non-Singular Matrix (S). The S matrix is used to scramble messages. Example:

$$S = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5. Calculate Public Key
The public key is calculated as:

$$G^1 = SGP$$

The private key consists of: S, G, P and a parity check matrix H.

Encryption Process

- Dividing the Message into 4-bit Blocks. Since Hamming(7,4) works with 4-bit blocks, the message is divided as follows: English: $m_1 = [0 \ 1 \ 0 \ 1]$
 $m_2 = [1 \ 0 \ 1 \ 0]$
- Encode Using Public Key. For each m_{imi} block, calculate the code word using:

$$c = mG'$$

- Add Error. To improve security, the error vector is added.

$$y = c + e$$

Example: English: $= [0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0]$

Thus, the ciphertext sent is:

$y1=[0\ 1\ 0\ 1\ 0\ 0\ 1]$
 $y2=[1\ 0\ 1\ 0\ 0\ 1\ 1]$

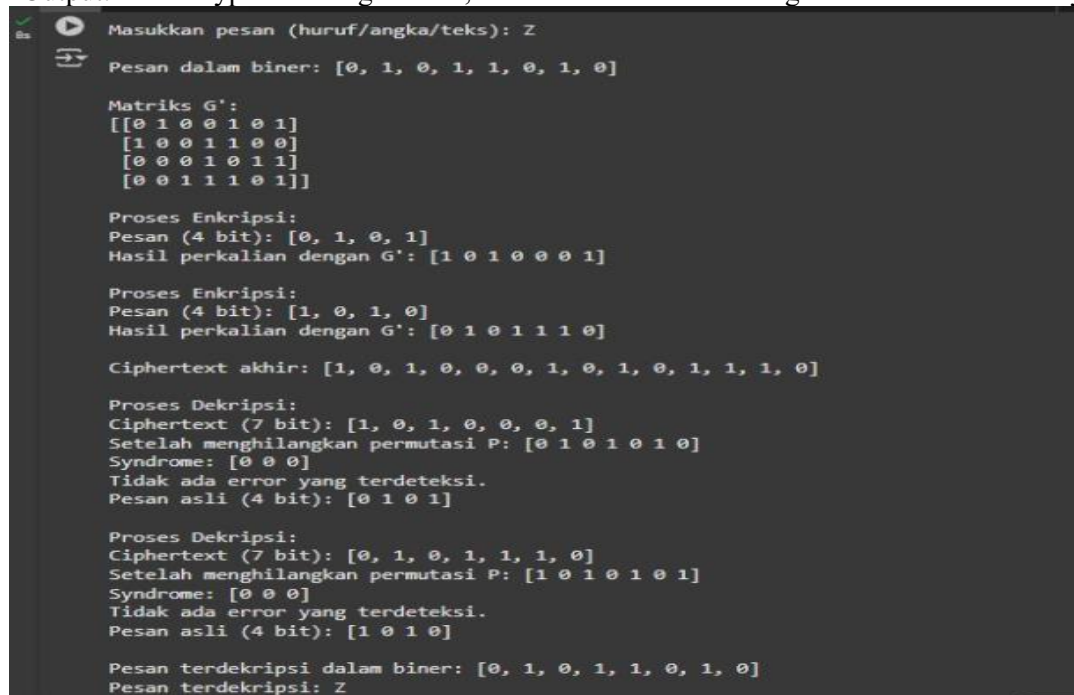
Process Description

- Receiving Ciphertext The recipient obtains: $y1=[0\ 1\ 0\ 1\ 0\ 0\ 1]$
 $y2=[1\ 0\ 1\ 0\ 0\ 1\ 1]$
- Remove Permutations. Using the inverse of P:
 $y' = yP^{-1}$
- Correcting Errors. Use the parity check matrix H to detect errors: $s=y'HT$. If $s \neq 0$, this indicates an error, and the error position is corrected.
- Decode Message. Using the inverse of S, take the original message: $m=c'S^{-1}$
 The final result is: $m=[0\ 1\ 0\ 1\ 1\ 0\ 1\ 0]$ which is the binary representation of "Z".

Testing With Python

Hamming Program Process and Results Brief Explanation:

- Input & Conversion. The user enters a text character ("Z"). The text is converted to ASCII binary format.
- Encryption. The message is divided into 4-bit blocks. Each block is encoded using a generator matrix (G'), producing a 7-bit ciphertext.
- Description. The ciphertext undergoes permutation elimination. This syndrome is calculated to detect and correct errors. The corrected data is converted back into 4-bit blocks and then into the original text.
- Output. The decrypted message is "Z", which confirms that the algorithm worked correctly.



```

Masukkan pesan (huruf/angka/teks): Z
Pesan dalam biner: [0, 1, 0, 1, 1, 0, 1, 0]

Matriks G':
[[0 1 0 0 1 0 1]
 [1 0 0 1 1 0 0]
 [0 0 0 1 0 1 1]
 [0 0 1 1 1 0 1]]

Proses Enkripsi:
Pesan (4 bit): [0, 1, 0, 1]
Hasil perkalian dengan G': [1 0 1 0 0 0 1]

Proses Enkripsi:
Pesan (4 bit): [1, 0, 1, 0]
Hasil perkalian dengan G': [0 1 0 1 1 1 0]

Ciphertext akhir: [1, 0, 1, 0, 0, 0, 1, 0, 1, 0, 1, 1, 1, 0]

Proses Dekripsi:
Ciphertext (7 bit): [1, 0, 1, 0, 0, 0, 1]
Setelah menghilangkan permutasi P: [0 1 0 1 0 1 0]
Syndrome: [0 0 0]
Tidak ada error yang terdeteksi.
Pesan asli (4 bit): [0 1 0 1]

Proses Dekripsi:
Ciphertext (7 bit): [0, 1, 0, 1, 1, 1, 0]
Setelah menghilangkan permutasi P: [1 0 1 0 1 0 1]
Syndrome: [0 0 0]
Tidak ada error yang terdeteksi.
Pesan asli (4 bit): [1 0 1 0]

Pesan terdekripsi dalam biner: [0, 1, 0, 1, 1, 0, 1, 0]
Pesan terdekripsi: Z
  
```

Figure 1. Encryption

The process of encrypting and decrypting a message using an error correction code, possibly the Hamming Code method. The message entered is the letter "Z", which is converted to binary [0, 1, 0, 1, 1, 0, 1, 0]. Next, this message is encrypted with a generator matrix (G'), producing a ciphertext in 7-bit form with redundancy bits for error detection. In the decryption process, the ciphertext is checked using a syndrome to detect errors. If there are no errors, the message is returned to its original form. The final result shows that the message was successfully sent and received without any changes, proving the effectiveness of the error correction method used.

Goppa Program Process and Results

Code Overview:

1. Input & Conversion, The user enters a character ("Z"), which is converted to binary form.
2. Encryption Process, The message is divided into 4-bit blocks (k-bits). Each block is multiplied by the generator matrix (G'), producing a 7-bit coded message (ciphertext).
3. Decryption Process, The ciphertext is processed by removing the permutations applied during encryption. The original 4-bit message is extracted from the 7-bit encoded data. The decoded binary sequence is converted back to text ("Z"), verifying its correctness.

```

Masukkan pesan (huruf/angka/teks): Z

Pesan dalam biner: [0, 1, 0, 1, 1, 0, 1, 0]

Matriks G':
[[0 1 1 1 0 1 0]
 [1 1 0 1 0 0 0]
 [1 1 1 1 1 1 1]
 [0 0 1 1 0 0 1]]

Proses Enkripsi:
Pesan (k bit): [0, 1, 0, 1]
Hasil perkalian dengan G': [1 1 1 0 0 0 1]

Proses Enkripsi:
Pesan (k bit): [1, 0, 1, 0]
Hasil perkalian dengan G': [1 0 0 0 1 0 1]

Ciphertext akhir: [1, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 1]

Proses Dekripsi:
Ciphertext (n bit): [1, 1, 1, 0, 0, 0, 1]
Setelah menghilangkan permutasi P: [1 1 0 1 1 0 0]
Pesan asli (k bit): [0 1 0 1]

Proses Dekripsi:
Ciphertext (n bit): [1, 0, 0, 0, 1, 0, 1]
Setelah menghilangkan permutasi P: [1 1 1 0 0 0 0]
Pesan asli (k bit): [1 0 1 0]

Pesan terdekripsi dalam biner: [0, 1, 0, 1, 1, 0, 1, 0]
Pesan terdekripsi: Z

```

Figure 2.Decryption process

This figure shows the process of encrypting and decrypting a message using an error-correcting code. The message "Z" is converted to binary [0, 1, 0, 1, 1, 0, 1, 0] and encrypted using a generator matrix (G') to produce a ciphertext with redundant bits. The ciphertext is then checked in the decryption process, where permutations are removed to recover the original message. No errors are detected, so the message is successfully decrypted back to binary and converted to the character "Z", indicating that the encryption and decryption methods work well without any data loss.

4. CONCLUSION

The McEliece cryptosystem implementation using Hamming(7,4) codes demonstrated successful encryption and decryption, with the ability to effectively correct single-bit errors. The system is very efficient in terms of execution time and provides strong security against attacks. However, its limitations in handling multiple errors indicate the need for alternative error-correcting codes in more challenging environments.

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